The Signifier as Aperture

Toward an Optical Critique of Arbitrariness

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The Real is not the void alone but the persistence of continuity through the void.

Before language communicates, it localizes (verortet).

Abstract

This article proposes that the camera obscura should be understood not merely as an optical instrument but as a materialization of a much older philosophical structure: *projection*. From Plato's cave and Neoplatonic theories of emanation to German Idealism and Lacanian psychoanalysis, appearance has repeatedly been conceived as the effect of a mediating operation through which presence becomes visible only by passing through absence. The camera obscura provides a privileged model of this operation because it transforms an external world into an inverted image through a minimal cut in space.

Building upon Lacan's Graph I, the present study argues that signification itself possesses a projective structure. The signifier is reconsidered not as a conventional label attached to a pre-given object but as an aperture through which objects, meanings, and subjects become projectable. This perspective allows a reconsideration of Saussure's doctrine of arbitrariness. While signifiers remain arbitrary with respect to resemblance, they are not arbitrary with respect to the conditions of appearance and localization (*Erscheinung* and *Verortung*). Like the aperture of the camera obscura, the signifier does not determine the identity of what appears, yet it participates decisively in the constitution of the field within which appearance becomes possible. The signifier does not resemble the world; it participates in the *Verortung des Erscheinens*: the localization of appearance.

The argument subsequently connects projection, the mirror stage, proper names, and Lacan's theory of the gaze to a broader topological account of continuity and division. Through a dialogue with Dedekind's construction of the real numbers, the article proposes that continuity is not the absence of gaps but their effect. The Real is approached not as a hidden substance behind language but as the impossible consistency generated through symbolic cuts. From this perspective, the camera obscura emerges not as a metaphor for signification but as one of its earliest material models. The Real itself is approached as a form of

Aus-stand[1]: not a reality standing behind language, but a consistency that *stands out* of language precisely where language fails to coincide completely with itself.

Part I: Before the Camera Obscura or The World as Projection

The camera obscura was already there before the camera obscura.

Long before the seventeenth-century optical device transformed projection into a material apparatus, projection had already occupied a central place in philosophy, theology, mythology, and symbolic life. The device did not invent the structure; it rendered visible a problem that had accompanied human cultures for centuries.

From the perspective of ethnotopology[2], philosophy itself appears as only one manifestation of a broader projective structure. The forms through which a culture imagines the relation between appearance and reality, presence and absence, inside and outside, often reveal recurring topological operations that reappear in myths, dreams, rituals, architectures, and systems of thought alike.

Plato's cave already presents a theory of projection. The prisoners do not encounter things themselves but shadows cast upon a wall. Yet the cave is more than an epistemological allegory of illusion. It introduces a fundamental ontological problem: presence appears only through mediation.

Christianity radicalizes this structure through incarnation. The Word does not merely describe flesh; the Word becomes flesh. What was *projection* here assumes a sort of *embodiment*.

German Idealism transforms projection into dialectic. Thought and being, concept and object, Spirit and Nature become moments of a recursive process in which each emerges through the other.

Lacan inherits this lineage but reformulates it through signification. What German Idealism approached through dialectic, Lacan approaches through the signifier. Projection becomes linguistic, topological, and subjective at once.

The history of projection therefore does not culminate in the invention of the camera obscura. Rather, the camera obscura materializes a structure that philosophy had already been exploring and that cultures had long been inhabiting in myths, symbols, dreams, and ritual forms. The device was invented in the seventeenth century; the structure is much older. Plato's cave, Hegel's

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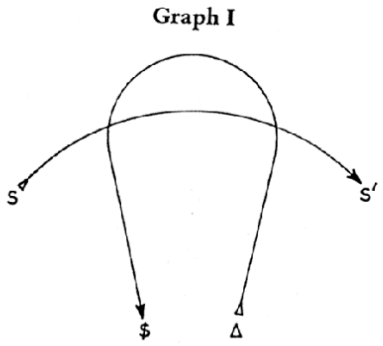
[1] The term *Aus-stand* exploits a productive ambiguity in German. *Ausstand* ordinarily denotes a strike, stoppage, or interruption of work. Heard etymologically, however, it may also be understood as *Aus-stand*: a standing-out or standing-forth from within a structure. The Real is approached here in both senses simultaneously: as an internal interruption of the Symbolic and as what becomes manifest precisely through this interruption.

[2] Check Don. Kunze's essay: Ethnotopology, URL: https://bpb-us-e1.wpmucdn.com/sites.psu.edu/dist/9/19778/files/2024/08/ethnotopology.pdf ; last accessed on 5.6.2026.

skull-bone, Persian/Arabic *ʿaks* عکس (reflection/inversion/image), the mirror stage, and Graph I all belong to the same genealogy of projection.[3]

Lacan's Graph I and the Recursion of Projection

The most striking feature of Lacan's teaching is its recursive structure. From his reading of Descartes' *cogito ergo sum* to his own radical aphorisms—*"there is no woman"*, or *"a signifier is what represents the subject for another signifier"*—we encounter a logic that turns back upon itself. Lacan's discourse never proceeds linearly but instead folds, loops, and reflects, so that every proposition gestures beyond itself and back into the system that produced it. This recursive movement is the kernel of Lacanian thought.



This recursive kernel is nowhere more clearly represented than in **Graph I** from *"The Subversion of the Subject and the **Dialectic** of Desire in the Freudian Unconscious."* [4]The arc from **S → S'** captures the relational character of signification: meaning emerges only in the movement from one signifier to another. The second arrow stretched from **Δ** (the locus of the Other and of desire) toward **$** (the divided subject) shows that the subject never appears directly but only as split, inscribed by the signifying chain.

Graph I, then, is not a *closed map* but a loop in which subject and signifier are reciprocally constituted. It is a schema of recursion itself: a signifier only has meaning by referring to another, and the subject only emerges as represented within this passage. Now we will see that there is more into this as the diagram contains a further, often overlooked, possibility.

The Topological Inversion

If we treat its arrows topologically—turning them inside-out—the symmetry of the graph comes to light. In this inversion, we no longer read: "a signifier represents the subject for another signifier", but rather: "***the subject relays the signifier to another subject, and is the very act of doing so***." This

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[3] Esmaeilpour Ghoochani, Iraj: "Cinema Is a Film: In the Phenomenology of the Spirit of Iranian Cinema," accessed June 4, 2026,
https://www.academia.edu/98962139/Cinema_is_a_Film_In_the_phenomenology_of_the_spirit_of_Iranian_cinema?sm=a&rhid=40572078975.

[4] *"The Subversion of the Subject and the Dialectic of Desire in the Freudian Unconscious"* is a lecture Lacan delivered at a colloquium of philosophers in Royaumont, France, from 19–23 September 1960, convened to explore the concept of the dialectic. The text engages not only with dialectical thought but also with "the subversion of the subject," a distinctly Hegelian notion. In Hegelian terms, the Master and Slave represent thesis and antithesis, whose interplay produces Identity as their synthesis—a dynamic Lacan interprets as the mechanics of subversion.

shift dramatizes the structure of speech. To speak is always to speak that could be to oneself and to the Other, and in doing so, both subject and signifier are altered.

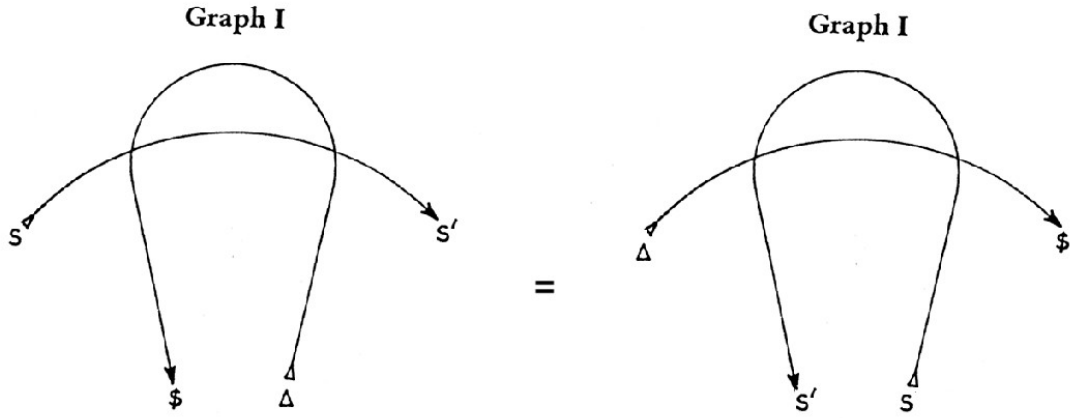
Graph I can be reread as a recursive dynamical system in which subject and signifier continuously function as one another's input. The subject is not a stable element occupying a fixed position within the system but the trajectory generated through its recursive iteration: Speech produces a transformed subject; the transformed subject produces a transformed signifier; the transformed signifier produces another transformed subject, and so forth.

Neither subject nor signifier remains identical to itself from one iteration to the next. Each becomes the condition for the transformation of the other. What persists is therefore not a substantial identity but a recursive process of non-coincidence.

Then, Graph I resembles a dynamical system more than a static structure. The subject does not occupy a point within the graph; it traces a path through it. One might say that the subject is not an element contained within the structure, but a trajectory generated by its repeated operation. This recursive movement also clarifies why the subject can never fully coincide with itself which of course is a very Hegelean point. Every act of speech returns to the speaking subject transformed. The output of one iteration becomes the input of the next. Signification therefore behaves less like a correspondence between fixed entities than like a process of continuous self-modification.

The signifier represents the subject for another signifier. Yet equally, the subject relays the signifier to another subject and becomes, in the very act of doing so, something other than it was. The graph therefore describes not a static relation but a circulation whose consistency depends upon perpetual transformation.

Such an inversion radicalizes Graph I: it reveals that the diagram always contained a hidden reciprocity, a symmetry between subject and signifier that mirrors the recursive logic of discourse:



Inside-out of Graph I. In Lacan's Graph I, the subject ($) appears as an effect of the passage from **S** to **S'**: *a signifier represents the subject for another signifier*. But if we treat the descending arrows as a continuous surface and turn it inside-out, the diagram licenses a complementary reading: ***the subject is what relays the signifier into the field of*** the ***Other***. Once touched by a signifier, the subject is divided; and in relaying that signifier to another subject, it is rewritten again. The two formulations are not opposites but two orientations of the same surface—each presupposes the other.

Symmetry, Speech, and German Idealism: The Forgotten Symmetry of Graph I

Let us sum up all these: Lacan's Graph I is usually interpreted in only one direction: *a signifier represents the subject for another signifier*. Yet its very structure suggests a symmetry that has often been forgotten. If the arrows are inverted, the subject itself becomes the vector of the signifier. In this **structural reciprocity**, signifier and subject stand in the same order: **each only exists insofar as it passes through the other.**

This symmetry recalls Descartes' *cogito*, where thinking was long treated as the proof of being. Just as thought and being mirrored one another in Descartes, subject and signifier mirror one another in Lacan. In both cases, certainty emerges not from linear deduction but from recursive self-reference. Speech itself shows us this structure: when we speak, (nevermind its address is ourselves and the Other subject,) and in the very act of speaking neither subject nor word remains the same.

This symmetry brings Lacan into direct dialogue with the tradition of German Idealism, where Descartes' problem of *thinking and being* was most radically reworked. Fichte, Schelling, and Hegel each proposed a doctrine of how concept and object, representation and being, stand to one another:

Fichte's subjective idealism: nothing possesses actuality apart from its relation to the activity of thought; reality is dependent on the self-positing "I."

Schelling's objective idealism: nature is accorded an independent status as precondition of the "I"; the subject is grounded in a reality external to it.

Hegel's absolute idealism: the antithesis between subjective and objective is resolved in the dialectical movement of Spirit, where both concept and being are continuously transformed.

Seen through Lacan's topological lens, this lineage takes on new force. The very reciprocity of subject and signifier dramatizes the dialectical interplay of concept and being; Word and Flesh, Spirit and the Bone![5]

Thus, spirit is the skull and vice versa. After speech, not only is the subject altered, but the signifier itself becomes something different. The structure of discourse mirrors the structure of dialectic: both are recursive processes of becoming.

For Hegel, the reconciliation of subjective and objective idealism occurs in the dialectical movement of Spirit. Spirit is not a static totality but a process of becoming, in which thought and being are mediated through their own negations. Lacan's Graph I, especially when read through its symmetrical inversion, provides a striking analogue of this movement. Yet the medium of transformation is different. In Hegel, concept and object transform one another through dialectical mediation. In Lacan, subject and signifier transform one another through speech, desire, and the circulation of jouissance. Every passage through the signifying chain leaves a remainder—objet petit a—which prevents closure and sets the circuit in motion again. The subject therefore persists not as a stable identity but as the trajectory generated through repeated encounters with its own constitutive non-coincidence. Both Hegel and Lacan think identity through movement, recursion, and negation; but where Hegel follows the labor of the concept, Lacan follows the detours of desire.

If subject and signifier emerge through projection, then projection is not merely a linguistic operation. We should expect to find homologous structures elsewhere. One such structure appears in the history of optics itself: the camera obscura.

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[5] Alluding to this sentence from Hegel: „*Die Wirklichkeit und Dasein des Menschen ist sein Schädelknochen!*“
The reality and Dasein of man is his skull-bone One might follow the logic of this essay to interchange the relation between the Spirit and its image: The Bone! The original Hegelian text commonly referenced for this proposition reads: *"Die Wirklichkeit und Dasein des Menschen ist sein Schädelknochen... Wenn das Sein als solches oder Dingsein von dem Geiste prädiziert wird, so ist darum der wahrhafte Ausdruck hiervon, dass er ein solches wie ein Knochen ist."*]*The reality and concrete existence of man is his skull-bone... When Being as such or thingness is predicated of Spirit, the authentic expression of this is that Spirit is a thing like a bone*.[(Hegel, G.W.F., *Phänomenologie des Geistes*, Ullstein [1973], pp. 180-202)

Cámara Oscura descrpita en Ars Magna Lucis Et Umbrae 1646 – Athanasii Kircher 1601 – 1680

The camera obscura does not illustrate signification. It materializes a structure that philosophy had already discovered in Plato's cave, German Idealism, and Lacan's Graph I: every appearance emerges through projection, and every projection becomes the source of another projection.

Kircher’s engraving above is crucial because it does not merely show an optical apparatus. It shows an inhabitant inside the apparatus. The observer does not encounter the outside world directly; he encounters a projection. But this projection is not the end of the process. Once perceived, it becomes the source of another projection: interpretation, naming, judgment, memory. The camera obscura therefore does not simply reproduce the world. It stages the passage from perception to signification. The decisive feature of Kircher's engraving is not the aperture but the observer. The projected image upon the wall is already a representation, yet for the observer it becomes the source of another operation: interpretation. The first projection transforms the external world into an image. The second transforms the image into meaning. The observer therefore inhabits a second camera obscura nested within the first. This recursive structure allows us to distinguish perception from conception. The image on the wall is not yet understanding. It must still be read. Yet this reading itself is another projection. Signification thus appears not as a transfer of information but as a chain of projections in which every image becomes the source of another image. The camera obscura therefore does not merely model vision. It models the recursive structure of signification itself.

Now, dear reader, something remarkable should become visible. Return for a moment to Lacan's Graph I. The machine before us is no longer merely an optical device. It is Graph I made wood, stone, and light. The world outside corresponds to the field from which signification emerges. The aperture performs the operation of selection and mediation. The projected image appears where meaning begins to take shape. Yet the true surprise stands quietly inside the chamber: the observer.

Without the observer, we have only optics.

With the observer, we have psychoanalysis.

For the projected image does not stop at the wall. It enters another circuit. It is perceived, interpreted, named, remembered, judged. The image becomes the source of another projection. The first camera obscura projects the world onto an image. The second projects the image into meaning.

Not exactly but in the same vain, speech cuts the subject, but in passing through the subject, the signifier itself is reconfigured. In other words, we can see the subject who cuts the signifier as a (stand-alone, quasi-automaton—an 'un-dead') subject, processing and relaying meaning. If not, "*what are poets for*?" (*Wozu Dichter*?') [6]

Inversion appears in more than one register. In optics it appears geometrically, as in the upside-down image of the camera obscura. In algebra it appears through reciprocal relations such as y = 1/x. In recursion it appears when a structure returns to itself only through transformation. The following examples should therefore not be read as mathematical curiosities but as different manifestations of the same operation: identity maintained through a passage into otherness.

Inversion: The Hidden Logic of Projection

The camera obscura does more than project. It inverts. The image appearing on the interior wall is turned upside down. What was above appears below. What was left appears right. It presents the world through inversion. This detail is often treated as a merely optical consequence of the aperture. Yet inversion may be the deepest lesson of the camera obscura.

For the history of philosophy repeatedly returns to structures that become intelligible only through reversal. To remember: Plato's cave inverts reality and appearance. Christianity inverts transcendence through incarnation. Hegel's dialectic proceeds through determinate negation and reversal (*Umkehrung*)[7].

The message returns from the Other in inverted form. Even the present study began by inverting Lacan's Graph I, revealing a hidden symmetry between subject and signifier.

The camera obscura therefore teaches a lesson that extends beyond optics. The Arabic and Persian word ʿ**aks (عکس)** which literally stands for inversion,

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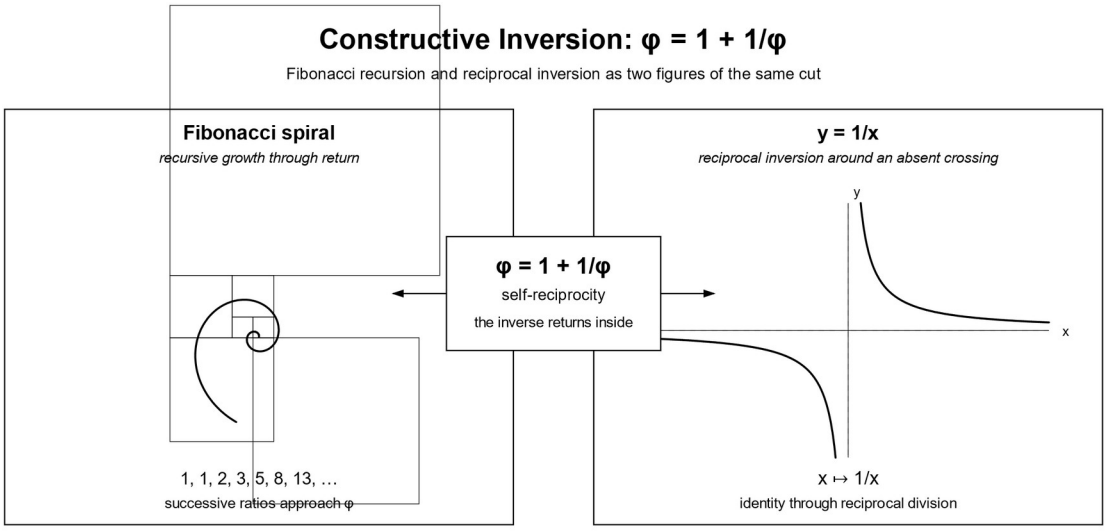
[6] In the age of AI, Heidegger's question—Wozu Dichter?—acquires renewed urgency. Machines can recursively transform signifiers, but poets dwell at the aperture where signification first becomes possible. If ordinary speech inhabits the projection, poetry remains closer to the cut through which language and being enter into relation. The poet does not merely manipulate signifiers but remains exposed to the event of signification itself.

[7] Cf. Dieter Redlich, *Die Wende im Bild* (Bern: Peter Lang, 1999). Against interpretations that treat Marx's references to Hegelian "*Umkehrung*" (upside-down) as merely metaphorical, Redlich investigates inversion as a precise philosophical operation. In this respect, his work is of particular interest here, since the camera obscura, reflection (*ʿaks*), dialectical reversal, and Lacanian signification all depend upon forms of inversion that are constitutive rather than accidental.

already preserves this intuition. An image is not simply a copy of the world but a reversal, a reflection, a turning-back. The photograph does not merely preserve reality; it produces a transformed relation to reality through inversion. From this perspective, inversion becomes a general principle of signification: The signifier is not the object; The image is not the source; The subject is not identical to itself, and so on…

Meaning emerges through a reversal that prevents any element from coinciding completely with its origin. Mathematics offers surprisingly similar examples. The hyperbola y = 1/x is generated through reciprocal inversion. The two branches remain separated by asymptotes they can endlessly approach but never cross: Identity persists through division. Continuity is subjected to a fundamental cut. Likewise, the Fibonacci sequence generates growth through recursive return. Each term arises from a relation to preceding terms rather than from an independent origin. The sequence converges upon the golden ratio without ever arriving there. The limit remains constitutively absent.


Constructive Inversion: φ = 1 + 1/φ
Fibonacci recursion and reciprocal inversion as two figures of the same cut
Fibonacci spiral
recursive growth through return
1, 1, 2, 3, 5, 8, 13, …
successive ratios approach φ
φ = 1 + 1/φ
self-reciprocity
the inverse returns inside
y = 1/x
reciprocal inversion around an absent crossing
y
x
x ↦ 1/x
identity through reciprocal division

Both figures stage inversion as construction: the curve by reciprocal division, the spiral by recursive return toward φ.

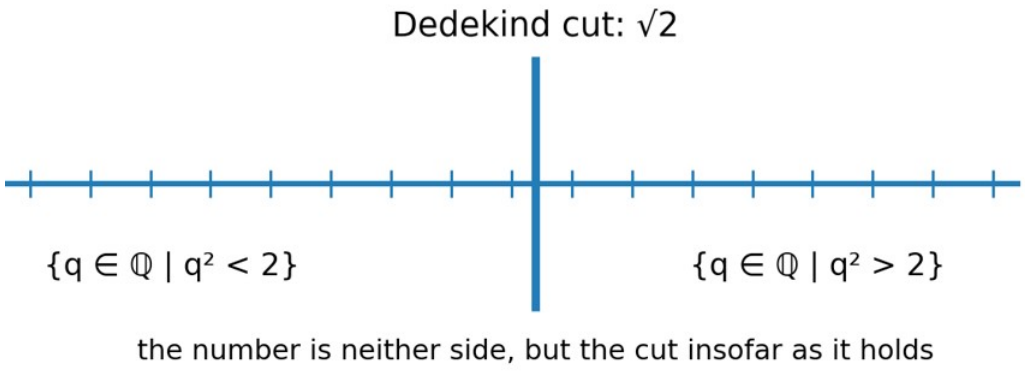
In the golden ratio, $\phi=1+1/\phi$, the inverse is not external to the structure but returns inside it. This is the mathematical figure of what the article calls constructive inversion: continuity emerges not by excluding the cut, but by folding inversion into its own law. Lacan's use of the Fibonacci series in Seminar XVI[8] similarly links recurrence, objet petit a, and the divided subject through an asymptotic structure that never closes into a perfect whole.

Lacan's subject behaves similarly: it includes its own inversion as an induction puzzle. The subject never encounters itself directly. It encounters itself only through signifiers returning from elsewhere, transformed by their passage through the Other. Every act of speech therefore contains a moment of inversion. The speaking subject returns to itself altered.

[8] "Fibonacci Series," Learn Lacan, accessed June 6, 2026, https://www.learnlacan.com/concepts/fibonacci-series/.

This recursive structure resembles the logic of induction puzzles. No single signifier produces a new subject, just as no single grain produces a heap[9], no single removed hair produces baldness, and no single term produces the Fibonacci limit. Yet with each iteration something shifts. The transformation cannot be localized positively within the sequence itself. One suddenly discovers oneself on the other side of a threshold that was never encountered as such.

The optical model of Camera Obscura, hence, becomes rigorous when placed beside Dedekind's construction of the real numbers. Dedekind's *Stetigkeit und irrationale Zahlen* [10](1872) does not treat irrational numbers as mysterious hidden points inserted into preexisting gaps. A real number is constructed through a cut, a *Schnitt* (*slot*), in the ordered field of rational numbers (Dedekind 1872).



Dedekind cut: the real number is the cut insofar as it holds continuity.

For example, √2 is not first a hidden point discovered somewhere between rationals. It is the cut separating {q ∈ ℚ | q² < 2} from {q ∈ ℚ | q² > 2}. The number is therefore neither side nor an external filler, but the operation of division itself insofar as the division holds. This changes the ontology of continuity. Continuity is no longer what exists before division and later becomes fractured. Continuity emerges through division.

This was the missing mathematical core of the camera obscura argument. The aperture matters not because it contains the image but because it produces the conditions of projection. The Saussurean bar matters not because it transports a signified but because signification emerges through its operation. Dedekind radicalizes the same structure: the cut does not destroy continuity; it constructs it.

At this point Lacan becomes legible in a new way. The Real is not empirical reality and not hidden fullness behind language. The Real emerges from the impossibilities or inconsistencies in the Symbolic, as Lacan repeatedly formulates

[9] "(2) Réel," accessed June 6, 2026, https://www.academia.edu/168304897/R%C3%A9el.

[10] Richard Dedekind, *Stetigkeit und irrationale Zahlen*, with University of Michigan (Braunschweig : F. Vieweg, 1892), http://archive.org/details/stetigkeitundir00dedegoog.

in different ways in the late seminars. It is only through the Symbolic that one is introduced to the Real, because without an inside there is no outside. The Real stands out of language in a kind of *Aus-stand*: it stands outside, but also marks the point where functioning stops. The Real appears where the Symbolic fails to coincide fully with itself, yet nevertheless continues to hold.

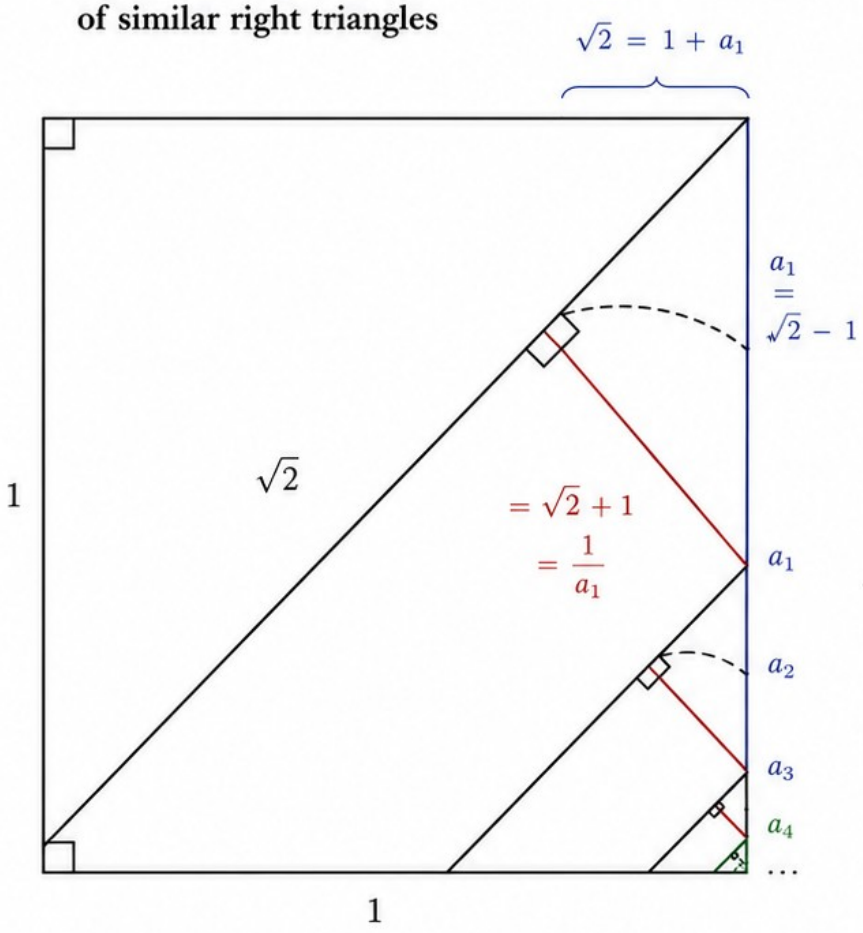
Thus and again, the Real is not the void alone, but the persistence of continuity through the void. The Symbolic stitches this impossible continuity together; the Imaginary experiences it as coherent form. As we will see, this triadic formulation will later return to the mirror stage and the proper name.

Lacan's appeal to the Fibonacci sequence may be understood as a special case of a more general principle. The essential feature is not the Fibonacci numbers themselves but the persistence of identity through recursive non-coincidence. A structure returns to itself only by passing through an operation that transforms it.

Dedekind's irrational numbers reveal the same logic. The number is never encountered directly but emerges through an endless refinement of a constitutive cut. Identity is not located in any particular approximation but in the persistence of the partition through successive transformations. The subject behaves similarly. It is not a positive element hidden behind signifiers but the consistency generated by their recursive passage. The subject persists because it never fully coincides with itself.

What Fibonacci recursion, continued fractions, Dedekind cuts, and Lacanian subjectivity share is therefore not a particular numerical formula but a common topology: identity generated through recursive division.

$\sqrt{2}$ from recursive construction of similar right triangles



1. First step

In the unit square, the diagonal has length $\sqrt{2}$.

Subtract 1 from $\sqrt{2}$:

$$a_1 = \sqrt{2} - 1 \approx 0.41421356\ldots$$

Thus $\sqrt{2} = 1 + a_1$.

Invert a_1:

$$\frac{1}{a_1} = \sqrt{2} + 1 \approx 2.41421356\ldots$$

The number returns, transformed.

Relation:

$$a_1 = \sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1}$$

2. Recursive construction

Use a_1 as the leg on the right side to build the next similar right triangle on the diagonal.

This gives the next remainder:

$$a_2 = \frac{1}{\frac{1}{a_1} - 2} = \frac{a_1}{1 + a_1} \approx 0.17157288\ldots$$

Invert:

$$\frac{1}{a_2} = 2 + a_1 \approx 2.17157288\ldots$$

Repeat indefinitely:

$$a_{n+1} = \frac{a_n}{1 + a_n} \qquad (n \geq 1)$$

Each new triangle is similar to the previous one. The remainders a_n decrease to 0.

3. Fixed-point (self-reciprocity)

The number $a = \sqrt{2} - 1$ is a fixed point of the map $a \mapsto \frac{a}{1+a}$:

$$a = \frac{a}{1 + a} \iff a^2 + a - a = 0 \iff a^2 + 2a - 1 = 0$$
$$\iff a = \sqrt{2} - 1.$$

Equivalently:

$$a = \frac{1}{2 + a} \iff \sqrt{2} - 1 = \frac{1}{2 + (\sqrt{2} - 1)}$$

4. Connection with the golden ratio φ

The golden ratio satisfies

$$\varphi = 1 + \frac{1}{\varphi}$$

Our number satisfies

$$\sqrt{2} - 1 = \frac{1}{2 + (\sqrt{2} - 1)}$$

Both are self-referential (fixed-point) relations of the form

$$x = c + \frac{1}{x} \qquad \text{(for a constant } c\text{)}$$

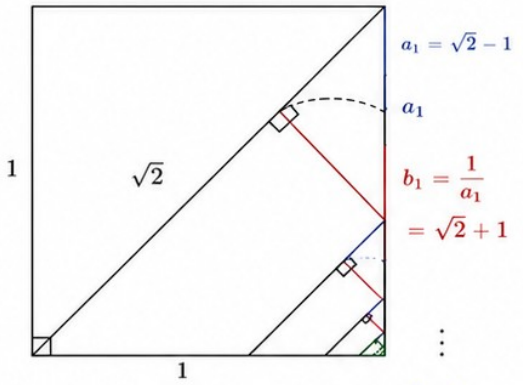
Number	Equation	Constant c	Value
φ	$x = 1 + \frac{1}{x}$	1	1.6180339...
$\sqrt{2} - 1$	$x = 2 + \frac{1}{x}$	2	0.41421356...

The same recursive inversion structure underlies both $\sqrt{2}$ and φ.
It is the algebraic signature of their deep resemblance.

Recursive Inversion: A General Principle for Irrational Numbers

Not every irrational number satisfies the same fixed equation.
What is general is the procedure: by subtraction, inversion, and renewed approximation, every irrational number is approached through an infinite recursive process. In special cases the pattern stabilizes; in general it does not.

A. Example 1: $\sqrt{2}$ (a quadratic irrational)



1. Start with the unit square. The diagonal has length $\sqrt{2} = 1 + a_1$.
2. Subtract 1: $a_1 = \sqrt{2} - 1$.
3. Invert: $b_1 = \frac{1}{a_1} = \sqrt{2} + 1$.
4. Use b_1 to build the next right triangle on the diagonal.
5. Repeat indefinitely.

The recursion stabilizes with a constant pattern

$a_{n+1} = \frac{1}{2 + a_n}$ $(a_0 = 1)$

Fixed point: $a = \frac{1}{2+a} = \sqrt{2} - 1$

Thus $\sqrt{2} = 1 + a$.

Continued fraction (periodic):

$$\sqrt{2} = [1;\overline{2}] = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}$$

Special feature: the coefficients repeat (period 1).
This is typical of quadratic irrationals.

B. General principle (valid for every irrational number)

Every irrational number α has a unique continued fraction expansion

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + + \cdots}}}$$

$a_0 \in \mathbb{Z}, \quad a_k \in \mathbb{N} \quad (k \geq 1)$

This is equivalent to the recursive process

$$r_0 = \alpha, \quad a_0 = \lfloor r_0 \rfloor, \quad r_1 = \frac{1}{r_0 - a_0}$$

$$r_{n+1} = \frac{1}{r_n - a_n}, \quad a_n = \lfloor r_n \rfloor \quad (n \geq 1)$$

What is general:

- At each step we subtract an integer part a_n (a "cut").
- We invert the remainder r_n (a "return").
- We iterate indefinitely.
- Rational numbers terminate (the remainder becomes 0).
- Irrational numbers never terminate.

Examples	$\sqrt{2}$ $=[1;\overline{2}]$	$\sqrt{3}$ $=[1;\overline{1,2}]$	$\phi = \frac{1+\sqrt{5}}{2}$ $=[1;\overline{1}]$	π $=[3;7,15,1,292,\ldots]$
a_0	1	1	1	3
a_1	2	1	1	7
a_2	2	2	1	15
a_3	2	1	1	1
a_4	2	2	1	292
a_5	2	1	1	1
⋮	⋮	⋮	⋮	⋮
	Periodic continued fractions (patterns stabilize)			Non-periodic continued fraction (no simple repeating pattern)

Dedekind allows us to generalize Lacan's Fibonacci intuition into a theory of identity generated through recursive division.

The subject is constituted precisely through this impossible passage. Like the irrational number emerging from a Dedekind cut, it appears not as an element contained within the chain but as the consistency generated by its recursive division. The subject therefore does not merely undergo inversion; it is the persistence of identity through inversion. Every return is a return as other.The camera obscura makes this visible.

Projection is inversion.

Signification is inversion.

Subjectivity is inversion.

The world appears only after passing through a cut that turns it inside out.

The recursive inversion visible in √2 and φ should not be mistaken for a special property of particular numbers. It is the visible manifestation of a more general topological operation. In quadratic irrationals, the operation appears algebraically through self-similar recursive equations. In circles, however, it appears geometrically.

The examples of √2 and φ may suggest that recursive inversion belongs only to certain mathematical structures. Aristotle's wheel reveals a more general possibility. Here inversion no longer appears algebraically but topologically. The paradox emerges because difference is folded into continuity. What Dedekind localizes as a cut, Aristotle distributes as a fold.

Aristotle's wheel reveals this hidden structure.[11] The paradox emerges because unequal circumferences are forced to advance as one. Difference is not eliminated but folded into continuity. As the circle is approximated by polygons, the discontinuities do not vanish. They lose their location and become distributed throughout the motion. The gap persists as a non-localizable remainder. The same logic underlies Dedekind's construction of irrational numbers. The irrational is never reached directly; it persists as the remainder generated by an endless process of localization.

The wheel therefore provides for circles what the Dedekind cut provides for irrational numbers. Both reveal that continuity is not primary. Continuity is an effect produced by a structured gap. In the Dedekind cut, the gap is progressively localized. In Aristotle's wheel, it is progressively delocalized. In both cases, identity persists only through a fold that prevents complete coincidence.

Why does this matter for language? Because the signifier functions in an analogous manner. It neither transmits a pre-existing identity nor creates one ex nihilo. Like the aperture, the cut, and the fold, it generates continuity through division.

The genius of Dedekind's cut, Saussure's bar, and Lacan's barred subject lies in a common discovery: the void and the division are not obstacles to continuity but its very condition.

At first glance, a cut appears as a lack. It separates, interrupts, and prevents coincidence. Yet Dedekind showed that continuity itself is generated through a cut. The irrational number does not fill the gap; the gap holds. Likewise, Saussure's bar separating signifier and signified is not a wall standing between two already constituted entities. It is the very operation through which signification becomes possible.

Lacan radicalizes this insight. The barred subject ($) is not a complete subject subsequently wounded by division. The bar is constitutive. Subjectivity emerges through the impossibility of coinciding fully with itself.

The remarkable consequence is that the bar and the void cease to be opposites. The bar gives the void a structure; the void gives the bar its efficacy. Dedekind's cut, Saussure's bar, the aperture of the camera obscura, and the Lacanian subject all reveal the same paradoxical principle: continuity is constructed through division.

What appears as absence is an operation. What appears as interruption is a condition of persistence. The cut does not threaten continuity. The cut is what allows continuity to hold.

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[11] Cf. one of my former essays: “The Gap as Fold Aristotle’s Wheel and the Temporal Twist of Space,” accessed June 6, 2026, https://www.academia.edu/166193540/The_Gap_as_Fold_Aristotles_Wheel_and_the_Temporal_Twist_of_Space.

The Observer in the Chamber

Start with:

For centuries commentators looked at the aperture...

Then:

The observer. The forgotten inhabitant of the machine.

Then:

Now, dear reader...

Then:

Graph I was already a camera obscura.

Then continue.

In other words:

keep the poetry,
remove the duplication.

Most discussions of the camera obscura focus on the relation between the external object and its projected image. Yet Kircher's engraving contains a fourth element that is often overlooked: the observer standing inside the chamber.

This figure changes everything.

For centuries commentators looked at the aperture.

Others looked at the projected image.

Yet the most important element was quietly standing inside the chamber all along.

The observer.

The forgotten inhabitant of the machine.

Without him, the camera obscura remains a model of optics. With him, it becomes a model of signification.

The outside world does not enter the chamber. The tower does not enter. The tree does not enter. The landscape does not enter. Only light enters through a small aperture, and what appears upon the wall is already a projection. Yet even this projection is not the end of the process. For the observer, the image on the wall becomes the source of another projection: interpretation.

The observer therefore inhabits a second camera obscura nested within the first.

The external world is projected onto the wall. The projected image is then projected into meaning.

Perception becomes conception.

Seeing becomes reading.

The image becomes a signifier. Yet signification is not the final station of the journey. The image first becomes a signifier, the signifier becomes meaning, and meaning in turn becomes the place from which a subject may appear. The camera obscura therefore contains not one projection but at least three nested projections: the projection of the world into image, the projection of the image into meaning, and the projection of meaning into subjectivity.

This nested structure explains why the camera obscura is not merely a model of perception but a model of signification. Every projection becomes the source of another projection.

At this point the engraving begins to anticipate Lacan's Graph I. A signifier represents a subject for another signifier. Likewise, the projection on the wall does not terminate in itself. It becomes the source of another operation. Signification therefore possesses the same recursive structure that we encountered earlier in German Idealism and in Lacan's topology.

The observer inside Kircher's chamber is not yet the Lacanian subject. He merely occupies the place where such a subject may emerge. Later we shall discover that the mirror stage introduces a second projection machine, and that the proper name functions as the aperture through which a body becomes projectable as a self.

The observer never encounters the source directly. He encounters only its projection. Yet from this projection an entire world of meaning emerges. The question is no longer how the world becomes visible, but how visibility itself becomes meaningful.

Only now are we ready to return to Saussure.

"Tree" is not leafy and the word "River" is not wet.

The camera obscura teaches us that mediation is not an unfortunate obstacle standing between us and reality. Mediation is the very condition under which reality becomes visible. Likewise, language does not stand between human beings and the world as a veil hiding a preexisting presence. Language participates in the production of appearance itself.

The signifier therefore resembles the aperture in a very precise sense. It does not contain the object. It does not reproduce the object. It does not create the object. It modulates the conditions under which the object may appear.

At this point the optical model begins to illuminate a limitation in the usual interpretation of arbitrariness. The shape of the aperture may be arbitrary relative to the source. Yet the aperture is not neutral. Its size affects resolution. Its edge generates diffraction. Its thickness influences transmission. Its material characteristics affect the image even when they do not determine the identity of the source.

The same may be said of signifiers.

A word does not need to resemble its referent. Yet words are not interchangeable without remainder. They possess rhythm, sound, visual form, historical sedimentation, cultural memory, and unconscious associations. They orient the body differently.

Jorge Luis Borges once remarked upon the curious smoothness of the word "moon." The observation should not be understood as proof that words secretly contain their objects. Rather, it points toward a subtler phenomenon. The signifier bends our approach to the object. The signifier modulates experience.

Something similar may be said of the Arabic القمر. Within the imaginary geography of the Arabic night, the visual contour of the script itself seems to participate in the lunar atmosphere surrounding the word. Whether this participation is objective matters less than the fact that subjects repeatedly experience such resonances.

Modern cognitive science has begun to encounter related phenomena. The Bouba-Kiki experiments demonstrate that speakers from different linguistic backgrounds tend to associate rounded sounds with rounded forms and angular sounds with angular forms. Such findings do not abolish arbitrariness. They reveal its limits. The signifier remains contingent, yet contingency is not equivalent to indifference.

One might therefore propose the following revision:

The signifier is arbitrary with respect to resemblance.

The signifier is not arbitrary with respect to modulation.

This distinction may appear modest, but its consequences are considerable. Once the signifier is understood as an aperture rather than a container, a new series of questions emerges. If the signifier functions like an aperture, what exactly appears through it? If language is structurally analogous to projection, what is being projected? Is it meaning? Is it the object? Or is it something else altogether?

The next step will suggest that what first appears through the signifier is neither the object nor the meaning of the object, but the subject itself. Before language communicates, it localizes. Before it informs, it positions. Before it signifies, it looks.

The history of modern linguistics begins with a gesture of suspicion. Saussure asks us to distrust the apparent intimacy between words and things. The word "tree" is not leafy. The word "river" is not wet. Nothing in the sound of a word guarantees a natural relation to the object it designates. Language functions because signs differ from one another, not because they resemble the world. Few discoveries have been more important for the human sciences.

Yet every foundational discovery casts a shadow. Once the signifier was liberated from resemblance, it gradually became detached from nearly everything else as well. Sound, rhythm, graphic form, bodily gesture, contour, and affect receded into the background. The signifier became increasingly abstract, almost weightless. It designated, but it no longer seemed to touch.

This article begins from a simple optical observation that may help us return materiality to the signifier without abandoning Saussure's central insight.

Anyone who has walked beneath trees during a solar eclipse has already witnessed the phenomenon. Hundreds of tiny images of the sun suddenly appear scattered across the ground. Each opening between the leaves functions as a miniature camera obscura. The surprising fact is that these projections do not inherit the accidental geometry of the openings through which they pass. A triangular gap does not project a triangular sun. A jagged opening does not project a jagged sun. What appears upon the ground is once again the solar disc itself.

The source reconstructs itself.

At first glance this seems to provide a perfect vindication of Saussure. The aperture appears arbitrary. Whether the opening is triangular, circular, square, or irregular, the same source survives.

Likewise the moon remains the moon whether it is called moon, Mond, lune, or القمر.

The signifier would seem to be little more than a contingent passageway.

Yet this conclusion overlooks the most important element in the experiment.

The aperture may not determine the identity of the source, but it determines the very possibility of projection. Without the aperture there is no image. If the opening becomes infinitely large, projection disappears. If the opening is completely sealed, projection disappears again. The image emerges only through a measured interruption.

The aperture therefore occupies a curious position. It is neither the source nor the image. It belongs fully to neither side. Yet without it neither side could enter into relation with the other.

This observation suggests a different way of approaching the signifier.

The signifier is not the object.

The signifier is not the meaning.

The signifier is the aperture through which the object becomes projectable as meaning.

The analogy should not be pushed carelessly. Language is not optics. Nevertheless, both systems appear to depend upon the same structural operation. In each case, appearance emerges through a cut.

Interlude: The Homunculus and the Cut

Before leaving Kircher's chamber, we should pause for a moment before its most enigmatic inhabitant.

The little observer standing inside the camera obscura immediately provokes a classical objection. If the image projected upon the wall requires an observer in order to become meaningful, who then observes the observer? And who observes that observer? The argument quickly generates an infinite regress known as the homunculus paradox: a little man inside the head looking at the image, another little man inside the first little man, and so on without end.

At first sight, this appears to be a problem of psychology. Yet it may belong to a much older family of paradoxes.

The structure is remarkably similar to the classical misunderstanding of irrational numbers. One imagines a gap between rational numbers and immediately asks: what occupies the gap? Another number. But then what occupies the gap around that number? Yet another number. The logic repeats indefinitely.

Richard Dedekind's breakthrough consisted precisely in refusing this question. The gap does not require another hidden occupant. The cut itself generates continuity.

Perhaps the homunculus paradox should be approached in the same way.

The observer does not require another observer hidden behind it. The observer is the cut itself.

What appears in Kircher's chamber is therefore not a miniature person but a structural position. The homunculus marks the point where projection folds back upon itself. The projected image becomes the source of another projection: interpretation, naming, judgment, memory. The observer is simply the name we give to this fold.

Now, dear reader, we may begin to suspect that the observer, the signifier, the subject, and the Dedekind cut occupy homologous positions.

The aperture is not merely a hole in the wall. It is a productive division through which an image becomes possible.

The observer is not merely a spectator standing before the image. It is a productive division through which meaning becomes possible.

Likewise, the Lacanian subject is not a complete being subsequently divided by language. The subject is the division itself insofar as the division holds.

The split is not in the subject.

The split is the subject.

At this point another possibility emerges. Perhaps words themselves should not be understood primarily as positive entities carrying meanings from one mind to another. Perhaps words function more like cuts in the continuum of experience. They partition, localize, and render projectable. Like the aperture of the camera obscura, they do not contain what appears through them. They make appearance possible.

The consequences of this proposition will accompany us throughout the remainder of this essay. For if the observer is a cut, if the subject is a cut, and if the signifier itself functions as a cut, then continuity, meaning, and identity may all depend less upon what fills the gap than upon the remarkable fact that the gap holds.

Part II

The Signifier as Aperture

Having spent some time inside Kircher's chamber, we may now return to Saussure with a different set of eyes.

The classical doctrine of arbitrariness begins with a liberation. Words need not resemble the objects they designate. The word "tree" is not leafy. The word "river" is not wet. Language functions because signifiers differ from one another, not because they imitate reality.

The camera obscura initially appears to confirm this position. The image projected within the chamber does not inherit the accidental geometry of the aperture. A circular sun may be projected through a triangular opening. The source survives despite variations in the opening.

At first glance, the aperture therefore seems irrelevant.

Yet this conclusion is too hasty.

The aperture may not determine the identity of the source, but it determines the very possibility of projection. Without the aperture, there is no image. If the opening becomes infinitely large, projection disappears. If it becomes completely sealed, projection disappears once more. The image emerges only through a measured interruption.

The aperture therefore occupies a peculiar position. It belongs neither to the source nor to the image. Yet without it neither source nor image could enter into relation.

Now, dear reader, we may begin to suspect that the signifier occupies a similar place.

The signifier is not the object.

The signifier is not the meaning.

The signifier is the aperture through which an object becomes projectable as meaning.

This proposition differs subtly from both naïve realism and radical constructivism. The signifier does not create the object. Nor does it merely transmit a pre-existing object unchanged. It modulates the conditions under which the object may appear.

The aperture therefore allows us to reformulate Saussure's doctrine in a more precise way.

The signifier is arbitrary with respect to resemblance.

The signifier is not arbitrary with respect to modulation.

The distinction is decisive.

A triangular aperture may still project a circular sun. Likewise, the moon remains the moon whether it appears as moon, Mond, lune, or القمر. The signifier need not resemble its referent.

Yet no aperture is neutral.

Its diameter affects resolution.

Its edge generates diffraction.

Its thickness influences transmission.

Its contour alters the image without determining the source.

Likewise, no signifier is neutral.

Every word possesses rhythm, sound, graphic form, historical sedimentation, affective coloration, and unconscious resonances. The signifier bends the approach to the object.

Jorge Luis Borges once remarked upon the peculiar smoothness of the word "moon." One should not understand this as proof that words secretly contain their objects. Rather, the signifier modulates our relation to what appears through it. The word does not hold the moon. It shapes a path toward it.

Something similar becomes visible in the famous Bouba-Kiki experiments. Subjects repeatedly associate rounded sounds with rounded

forms and angular sounds with angular forms. Such experiments do not abolish arbitrariness. They reveal its limits. The relation remains contingent, yet contingency is not indifference.

The body responds to contours.

The body responds to sounds.

The body responds to rhythms.

The body responds to letters.

At this point the camera obscura begins to reveal a deeper lesson. We usually imagine that words are positive entities carrying meanings from one mind to another. But perhaps this is precisely backwards.

The aperture does not contain the image.

The aperture makes the image possible.

Likewise, words do not contain meanings.

Words make meanings projectable.

This is why the signifier resembles the observer more than we first suspected. The observer and the aperture occupy homologous positions. The aperture mediates between source and image. The observer mediates between image and meaning. Both are productive cuts. Both introduce a division through which a new consistency emerges.

The observer, one might say, is a living aperture.

And perhaps words themselves belong to the same family.

A word is not merely a thing among things.

A word is a partition within experience.

A cut through continuity.

A localization.

A threshold.

A productive void.

Like the Dedekind cut, it divides and generates simultaneously.

The consequences of this proposition extend far beyond linguistics. For if words function as apertures, and if subjects emerge through analogous cuts, then identity itself may be less substantial than ordinarily imagined. The self would no longer be the hidden owner of language. It would be one of the effects produced by language's projective machinery.

It is here that the child enters the scene.

For before the child learns what things mean, the child must first learn where it stands among them. Before language communicates, it localizes. Before it informs, it positions. Before it signifies, it looks.

The next chapter will therefore return to a second projection machine: the mirror.

Part III

The Mirror as a Second Camera Obscura

At this point a new question emerges.

If the aperture makes objects projectable as meanings, how does a body become projectable as a self?

The answer leads us toward another projection machine.

The mirror.

Traditionally, Lacan's mirror stage is understood as the moment at which the child identifies with its own image. The child encounters a coherent form where previously only fragmented sensations existed. Through this image, an ego begins to emerge.

Yet viewed through the lens of projection, something even more fundamental becomes visible.

The mirror is a second camera obscura.

The difference is that the projected object is no longer a tree, a tower, or a landscape.

The projected object is the observer itself.

The observer standing inside Kircher's chamber now becomes the source of a new projection.

A body appears as an image.

An image becomes recognizable.

A recognizable image becomes inhabitable.

A new consistency emerges.

Now, dear reader, we may return for a moment to the forgotten inhabitant of Kircher's chamber.

Earlier we discovered that the observer should not be understood as a little person hidden behind perception. The homunculus was not another object but a constitutive cut, a point at which projection folds back upon itself. The observer was the place where an image became meaningful.

The mirror stage repeats exactly the same operation.

The child does not first possess a complete self and then discover its image. Just as the Lacanian subject does not preexist symbolic division, the self does not preexist its projection.

The image does not represent a finished identity.

The image generates a new consistency.

Here Dedekind's lesson returns unexpectedly. The irrational number does not exist before the cut and later become discovered. The cut generates the consistency of the number. Likewise, the self does not exist before projection and later become recognized. The projection generates the consistency of the self.

The mirror therefore does not reveal identity.

It produces identity.

But something is still missing.

An image alone cannot tell the child that the image belongs to it.

The image requires a signifier.

It requires an address.

It requires a name.

Here Peter Widmer's reflections on the proper name become decisive. The child does not merely encounter a reflected body. The reflected body is already embedded within a symbolic field. Others point, speak, address, and identify. The image is captioned before it is understood.

The child learns not merely to see.

The child learns to be seen.

And it is precisely here that the proper name enters.

The proper name is the aperture through which a body becomes projectable as a self.

The formula may appear surprising, yet it follows directly from everything we have established thus far.

The aperture does not contain the image.

The aperture makes the image possible.

Likewise, the proper name does not contain the self.

The proper name makes the self projectable.

A body becomes localized within the symbolic order through a signifier that appears to designate it. Yet the designation is productive rather than descriptive. The name does not simply identify a preexisting self. The name participates in the emergence of the self it names.

This is why names often exceed their apparent semantic function. They carry unconscious investments, family histories, forgotten desires, and symbolic destinies. Long before the child understands what a name means, the name already occupies a place within the desires of others.

The mirror image therefore never appears alone.

The image and the name arrive together.

The Imaginary and the Symbolic emerge together.

The image provides form.

The name provides localization.

The image says: "there is a whole."

The name says: "that whole is you."

At this point we may finally understand why the signifier appears phenomenologically as gaze.

The gaze is not another eye hidden behind perception.

Nor is it simply the biological act of looking.

The gaze emerges at the precise point where visibility and signification become inseparable.

The child experiences itself as visible through the field of the Other. The image is already traversed by signifiers. One sees oneself through being seen.

The gaze is therefore the experiential appearance of projection itself.

Just as the signifier functions as an aperture within language, the gaze functions as an aperture within visibility.

The child enters the mirror and discovers not merely an image, but a place within the symbolic order.

The observer has become observable.

The projection has become self-projecting.

And the camera obscura has quietly transformed itself into a subject.

Part IV

Dedekind, Continuity, and the Real

At this point the reader may object.

Everything in the preceding chapters appears to depend upon cuts, partitions, apertures, and divisions. The signifier divides. The mirror divides. The proper name divides. The observer itself was reinterpreted as a cut. Yet division ordinarily destroys continuity rather than generating it.

Why then does projection not collapse into fragmentation?

Why does the subject not simply shatter into disconnected pieces?

Why does meaning hold together at all?

The question leads us toward an unexpected ally: Richard Dedekind.

For centuries continuity was imagined as the absence of interruption. A line appeared continuous because no gap was visible within it. Division seemed secondary, accidental, and destructive. One first imagined continuity and then explained rupture as something occurring within it.

Dedekind reversed this intuition.

The irrational number is not a mysterious point later inserted into a pre-existing continuity. The cut itself generates continuity.

This is perhaps the most radical ontological gesture in modern mathematics.

Consider the famous case of √2.

The classical imagination pictures √2 as a hidden point occupying a gap somewhere among the rationals. Yet Dedekind's construction reveals something more surprising. The number emerges through a partition of the rational field itself:

$\{q \in \mathbb{Q} \mid q^2 < 2\}$

and

$\{q \in \mathbb{Q} \mid q^2 > 2\}$

The number is neither side.

Nor is it a third thing inserted between them.

The number is the cut itself insofar as the cut generates a new consistency.

The gap holds.

Continuity therefore appears in a completely new light.

Continuity is not what exists before division.

Continuity emerges through division.

Now, dear reader, we may return to the subject.

The barred subject ($) should not be understood as a complete self subsequently wounded by language. This is the psychological version of the old mathematical intuition. One imagines an intact unity and then explains division as a secondary accident.

Lacan's notation suggests something quite different.

The subject does not precede the bar.

The bar generates the subject.

Just as √2 does not precede the cut.

The cut generates √2.

The split is not in the subject.

The split is the subject.

At this point the Real becomes newly legible.

The Real is not a hidden substance concealed behind the symbolic order.

Nor is it a primordial fullness lost through language.

The Real appears where symbolic consistency depends upon a constitutive impossibility.

The Real is not what fills the gap.

The Real is the consistency of the gap itself.

This is why Lacan repeatedly approaches topology, knots, surfaces, and cuts. Mathematics does not provide decorative metaphors for psychoanalysis. Rather, psychoanalysis and mathematics appear to discover the same ontology from different directions.

Both discover that division can generate consistency.

Both discover that continuity can emerge through rupture.

Both discover that identity can persist through non-coincidence.

At this point several ancient paradoxes suddenly become intelligible.

Consider the Sorites paradox.

At which grain does a heap begin?

At which removed hair does baldness appear?

At which replaced plank does the Ship of Theseus become another ship?

The paradox emerges because we search for the transformation among the objects themselves.

We look for the decisive grain.

The decisive hair.

The decisive plank.

Yet the transformation does not occur in the objects.

It occurs in the constitutive cut organizing their continuity.

No single grain creates the heap.

No single removed hair creates baldness.

No final plank transforms the ship.

The transition occurs in an im-passable threshold.

Not merely impossible.

Im-passable.

The threshold cannot be occupied because it is itself the operation of passage.

One might say that the Sorites paradox and the homunculus paradox are distant relatives.

In the first case we search endlessly for the decisive object.

In the second we search endlessly for the decisive observer.

Both searches fail for the same reason.

They seek a positive occupant where there is only a productive cut.

The observer is not hidden behind observation.

The cut observes.

The heap is not hidden behind the grains.

The cut heaps.

The subject is not hidden behind the signifier.

The cut speaks.

And perhaps this is why the Real and the mathematical symbol ℝ stand so unexpectedly close to one another.

Not because psychoanalysis secretly reduces to mathematics.

But because both encounter a strange form of existence in which consistency emerges through division, continuity emerges through rupture, and identity survives only by failing to coincide completely with itself.

The Real is not the void alone.

The Real is the persistence of continuity through the void.

The Symbolic continuously stitches this impossible continuity together.

The Imaginary is the experience of this continuity as coherent form.

And the subject stands precisely where these three operations meet.

Part V

Letters, Lalangue, and the Body of the Signifier

At this point we may return to the question that originally motivated this investigation.

Are words truly arbitrary?

The question now appears in a different light.

Throughout this essay we have repeatedly encountered structures that function not as containers but as apertures. The camera obscura does not contain the image. It makes the image possible. The observer does not contain meaning. It makes meaning possible. The subject does not exist prior to symbolic division. The subject emerges through it. The Real itself appeared not as a hidden substance but as the consistency generated through a constitutive cut.

The signifier therefore no longer appears as a neutral vehicle transporting meanings from one mind to another. It behaves more like an aperture, a threshold, or a productive partition.

Yet if this is true, the materiality of the signifier can no longer be dismissed as irrelevant.

Classical structuralism often emphasized the arbitrariness of the sign in order to free language from naïve theories of resemblance. This gesture was necessary and remains indispensable. The word "tree" is not leafy.

The word "river" is not wet. No natural necessity binds signifier to referent.

And yet something continues to resist complete arbitrariness.

The body hears.

The body sees.

The body responds.

The body remembers.

Long before a word communicates a concept, it already possesses rhythm, contour, duration, texture, and resonance. The signifier arrives not only through meaning but through the body.

This is precisely what Lacan sought to designate through the notion of *lalangue*. Before language becomes a system of communication, it inhabits the body as sound, rhythm, accident, repetition, and enjoyment. The signifier is experienced before it is understood.

The child does not first learn meanings and later discover sounds.

The child first encounters sounds that leave traces.

Meaning arrives afterward.

At this point the letter acquires a new importance.

The letter is not merely the graphic representation of speech.

The letter is the visible body of the cut.

A letter partitions space exactly as a signifier partitions experience.

It divides and unites simultaneously.

It interrupts continuity while generating a new continuity of its own.

The Arabic notion of *ḥarf* reveals this with particular clarity. The word means both "letter" and "edge." The coincidence is not accidental. The letter is an edge. It is a border through which appearance becomes organized. The signifier and the cut meet visibly within the letter.

Now, dear reader, we may return once more to the camera obscura.

The aperture does not resemble the image it projects.

Yet neither is it irrelevant to that image.

Its contour matters.

Its edge matters.

Its materiality matters.

The aperture is arbitrary with respect to resemblance but not arbitrary with respect to modulation.

Perhaps the same must be said of language.

The word does not contain its object.

Yet neither is it indifferent to it.

Jorge Luis Borges once remarked upon the peculiar smoothness of the word "moon." One should not interpret this as a return to magical theories of language. Rather, it points toward the fact that signifiers possess their own material trajectories. They orient perception. They bend attention. They modulate experience.

The famous Bouba-Kiki experiments reveal a similar phenomenon. Subjects repeatedly associate certain sounds with certain forms. The relation is not necessary, yet it is not completely arbitrary either. Between absolute necessity and absolute contingency another domain begins to appear: the domain of modulation.

It is here that the work of Mansour Momeni and the Alifba project acquires a broader significance.

The letters are not merely carriers of meaning.

The letters participate in meaning.

Their form is not external to signification.

Their contour is not a disposable shell surrounding an abstract content.

The letter itself thinks.

Or more precisely: the letter localizes thought.

From this perspective, writing does not simply represent speech, just as speech does not simply represent reality. Every level functions as a projection of another level. Every projection becomes the source of another projection.

Graph I was already a camera obscura.

The camera obscura was already a Graph I.

The mirror repeated the camera obscura.

The proper name repeated the mirror.

The letter repeats them all.

For the letter is perhaps the smallest visible form of the cut.

A tiny aperture through which continuity becomes projectable.

And perhaps this is why mystical and poetic traditions repeatedly approached letters with such seriousness. The old intuition that "names descend from heaven" should not be dismissed too quickly. Not because words contain hidden essences, but because words participate in the very operation through which a world becomes thinkable.

The signifier does not stand opposite the world.

The signifier participates in the projection of the world.

The Real does not lie behind language.

The Real stands out through language where language encounters its own constitutive impossibility.

The Symbolic continuously stitches this impossible continuity together.

The Imaginary experiences it as coherent form.

And the letter marks the place where these three dimensions become visible at once.

A cut.

An edge.

A ḥarf.

A tiny aperture through which a world appears.

The Observer as Dedekind Cut

We may now return one final time to the little observer standing inside Kircher's chamber.

At first, the figure appeared to solve a problem. Someone had to observe the projected image. Someone had to transform perception into meaning. Yet the solution immediately generated another difficulty. If the image requires an observer, who observes the observer? And who observes that observer? The question rapidly produces the classical homunculus paradox: an infinite series of observers nested within observers.

The usual response is to reject the homunculus altogether. Modern thought often treats it as a naïve mistake, a relic of pre-scientific psychology. Yet perhaps the paradox should not be eliminated so quickly.

For the homunculus behaves remarkably like another figure we have already encountered: the irrational number.

The naïve imagination encounters a gap within continuity and immediately asks what occupies it. A hidden point. Another number. Something inserted between the two sides. Yet Dedekind demonstrated that this question already misunderstands the structure of the problem. The gap does not require an additional occupant. The cut itself generates continuity.

The homunculus paradox may be approached in exactly the same way.

Who observes the observer?

The question assumes that observation requires another observer standing behind it, just as naïve intuition assumes that continuity requires another positive element filling the gap. In both cases the search produces only bad infinity.

The observer of the observer.

The observer of the observer of the observer.

Another number.

Another number.

Another number.

The regress continues because one searches for an object where there is only a structural operation.

Dedekind's solution was to stop searching.

The cut is the number.

Likewise, perhaps we should stop searching for the hidden observer.

The observer is the cut.

This proposition transforms the entire camera obscura.

The observer is no longer a little person standing before the image. The observer becomes the point at which projection folds back upon itself. The projected image becomes the source of another projection: interpretation, memory, naming, judgment. The observer names this fold.

The observer is therefore not another object inside the chamber.

The observer is the chamber's internal aperture.

A living aperture.

A cut through which meaning becomes possible.

Now a surprising symmetry begins to emerge.

The aperture stands between source and image.

The observer stands between image and meaning.

The signifier stands between thing and signification.

The subject stands between signification and desire.

Each occupies the same structural position.

Each is neither one side nor the other.

Each appears as a productive division.

Each generates continuity by preventing continuity from coinciding completely with itself.

At this point we may finally understand why the subject cannot be located positively within the symbolic order. The subject does not exist on one side of the cut or the other. Just as $\sqrt{2}$ is neither side of a Dedekind partition, the subject is neither fully inside language nor fully outside it.

The subject is the cut insofar as the cut holds.

The observer is the cut insofar as the cut observes.

The signifier is the cut insofar as the cut signifies.

And perhaps this allows us to reformulate the relation between gaze and signifier in a more rigorous way.

The gaze is not another eye hidden behind perception.

The gaze is what appears phenomenologically when a cut becomes operative within visibility.

Likewise, the signifier is what appears structurally when a cut becomes operative within language.

The signifier and the gaze therefore do not simply resemble one another. They are two manifestations of the same operation viewed from different sides.

One may therefore risk the following proposition:

The signifier appears phenomenologically as gaze.

Not because language is secretly visual, nor because vision is secretly linguistic, but because both emerge through the same projective structure.

The camera obscura made this structure visible.

The mirror stage localized it.

Dedekind formalized it.

Lacan gave it a subject.

And perhaps the history of philosophy has been circling around it from the very beginning.

The observer was never hidden in the chamber.

The observer was the cut.

Conclusion

The World as Aperture

We may now return to Saussure.

The question that initiated this investigation was deceptively simple: are words arbitrary?

At first sight the answer seemed obvious. The word "tree" is not leafy. The word "river" is not wet. Nothing in the sound of a signifier guarantees a natural resemblance to the object it designates. Modern linguistics was built upon this discovery.

Yet as we followed the path from Plato's cave to German Idealism, from Lacan's Graph I to Kircher's camera obscura, from the mirror stage to Dedekind's cut, the question itself gradually transformed.

The issue was never resemblance.

The issue was projection.

The camera obscura taught us that the aperture does not resemble the image it helps produce. Yet neither is it irrelevant to that image. It participates in the conditions of appearance. The aperture is arbitrary with respect to resemblance, but not arbitrary with respect to modulation.

Perhaps the same must be said of signifiers.

Words do not mirror things.

Words do not simply label things.

Words participate in the projection of things.

A signifier is not a container of meaning. It is an aperture through which meaning becomes possible.

The consequences of this proposition extend beyond linguistics. Throughout this essay we repeatedly encountered structures that behaved in the same way. The aperture of the camera obscura, the observer within the chamber, the mirror image, the proper name, the barred subject, the Dedekind cut, objet petit a, and even the letter itself all appeared as productive divisions. None functioned as positive substances. Each generated continuity by introducing a constitutive non-coincidence.

The observer was not a hidden spectator.

The observer was the cut.

The subject was not a wounded unity.

The subject was the cut.

The Real was not what filled the gap.

The Real was the persistence of continuity through the gap.

Seen from this perspective, Lacan's famous distinction between the Symbolic, the Imaginary, and the Real acquires a new coherence.

The Symbolic continuously stitches continuity together.

The Imaginary experiences this continuity as coherent form.

The Real persists as the constitutive impossibility through which continuity holds.

One might say that the Symbolic sews, the Imaginary sees, and the Real insists.

The entire article may therefore be condensed into a single reversal.

Classical thought imagines continuity first and division afterward.

The present investigation suggests the opposite.

Division comes first.

Continuity emerges through division.

The aperture comes first.

Projection emerges through the aperture.

The cut comes first.

The subject emerges through the cut.

And perhaps this is why the history of philosophy repeatedly returned to caves, mirrors, shadows, projections, names, letters, knots, and numbers. These were never merely metaphors. They were encounters with a structure that appears whenever continuity becomes thinkable.

The camera obscura was already there before the camera obscura.

Plato's cave already knew it.

The mystics approached it through names.

German Idealism approached it through dialectic.

Dedekind approached it through mathematics.

Lacan approached it through the subject.

And perhaps every word still bears its trace.

For a word is not merely a sound.

A word is a tiny aperture.

A small and productive void.

A cut through which a world appears.

Coda

The Borromean Aperture

One final observation may help gather the threads of this investigation.

Throughout the article we repeatedly encountered a peculiar structure. The aperture of the camera obscura, the observer inside the chamber, the mirror image, the proper name, the signifier, the Dedekind cut, objet petit a, and the Lacanian subject all appeared as productive divisions. None functioned as positive substances. Each generated consistency through a constitutive non-coincidence.

The Borromean knot allows us to see why.

The Imaginary, the Symbolic, and the Real do not form a unity because they share a common center. Quite the contrary. Their consistency depends upon the absence of such a center.

Remove one ring and the entire structure falls apart.

The knot therefore possesses a topology remarkably similar to the structures encountered throughout this essay.

The Symbolic continuously stitches continuity together.

The Imaginary experiences this continuity as coherent form.

The Real persists as the impossibility preventing complete closure.

Yet none of these dimensions occupies the center.

The center is empty.

Or more precisely, the center functions like the aperture of the camera obscura.

A productive void.

A constitutive cut.

A place that does not exist positively and yet organizes everything around it.

Now, dear reader, we may return one last time to Dedekind.

The irrational number is not a hidden object occupying the gap.

The gap holds.

Likewise, the subject is not a hidden substance occupying the symbolic order.

The cut holds.

Likewise, the observer is not a little man standing behind perception.

The fold holds.

Likewise, the Borromean knot does not derive its consistency from a positive center.

The void holds.

This allows us to reformulate the central thesis of the article in its most condensed form.

The Real is not the void alone.

The Real is the persistence of continuity through the void.

The Symbolic is the stitching mechanism through which this impossible continuity is continuously re-established.

The Imaginary is the phenomenological experience of this continuity as coherent form.

The Borromean knot therefore appears as a topological expression of the same structure that Dedekind discovered mathematically, that Kircher materialized optically, and that Lacan formalized psychoanalytically.

The continuity of the line.

The continuity of the image.

The continuity of the subject.

The continuity of desire.

All depend upon a constitutive cut.

The world does not appear despite the aperture.

The world appears through the aperture.

And perhaps this is why the smallest unit of language—the letter itself—already contains the entire problem.

A ḥarf.

An edge.

A cut.

A tiny Borromean aperture through which continuity becomes visible.

Addendum

The Point of the Bāʾ

The preceding argument developed through philosophy, mathematics, optics, and psychoanalysis. Yet the structure it uncovered may be recognized within older mystical traditions as well.

A celebrated saying attributed to Imam ʿAlī states:

بالباء ظهر الوجود

Through the Bāʾ, existence became manifest.

The statement has often been interpreted esoterically. Yet read through the lens of projection, it acquires a striking topological resonance.

The Arabic letter ب consists of two elements: a horizontal stroke and a point beneath it.

If the horizontal line is taken to represent the continuity of being, the point beneath it occupies a peculiar position. It is neither the line itself nor something external to it. It appears as a minimal interruption, a localization, a singularity.

One might say: a cut.

The point functions much like the aperture of the camera obscura.

It does not contain existence.

It renders existence projectable.

Without the point, the line remains undifferentiated continuity. With the point, a letter emerges. A localization occurs. Appearance becomes possible.

In this sense, the mystical intuition approaches a structure we have encountered repeatedly throughout this essay.

The aperture of the camera obscura.

The signifier.

The proper name.

The Dedekind cut.

The Lacanian subject.

Each appears not as a positive substance but as a productive void through which consistency becomes visible.

The point of the Bāʾ may therefore be understood as a miniature figure of the entire argument.

Being does not appear despite the cut.

Being appears through the cut.

Or, in the language of the tradition:

بالباء ظهر الوجود

Through the Bāʾ, existence became manifest.